Return Loss and Mutual Coupling Coefficient of a Microstrip Printed Antenna Array Conformed on a Cylindrical Body for $TM_{01}$ Mode

Ali Elrashidi$^1$, Khaled Elleithy$^2$, Hassan Bajwa$^3$

$^1$Department of Computer Science and Engineering, University of Bridgeport, Bridgeport, CT 06604, USA
(aelrashi@bridgeport.edu)
$^2$Department of Computer Science and Engineering, University of Bridgeport, Bridgeport, CT 06604, USA
(elleithy@bridgeport.edu)
$^3$Department of Electrical Engineering, University of Bridgeport, Bridgeport, CT 06604, USA
(hbjwa@bridgeport.edu)

Abstract Curvature has a great effect on fringing field of a microstrip antenna and consequently fringing field affects effective dielectric constant and then all antenna parameters. A new mathematical model for return loss and mutual coupling coefficient as a function of curvature for two element array antenna is introduced in this paper. These parameters are given for $TM_{01}$ mode and using three different substrate materials RT/duroid-5880 PTFE, K-6098 Teflon/Glass and Epsilam-10 ceramic-filled Teflon.

Keywords Fringing field, Curvature, effective dielectric constant and Return loss (S11), mutual coupling coefficient (S12), Transverse Magnetic $TM_{01}$ mode.

1. Introduction

Microstrip antenna array conformed on cylindrical bodies is commonly used antennas in aircraft, millimeter-wave imaging arrays mounted on unmanned airborne vehicles, and antennas for medical imaging applications which may be required to conform to the shape of the human body [1]-[3]. Low profile, low weight, low cost and its ability of conforming to curve surfaces [4], conformal microstrip structures have also witnessed enormous growth in the last few years. Some advantages of conformal antennas over the planer microstrip structure include, easy installation (random not needed), capability of embedded structure within composite aerodynamic surfaces, better angular coverage and controlled gain, depending upon shape [5, 6].

While Conformal Antenna provide potential solution for many applications, it has some drawbacks due to bedding [7]. The two main disadvantages of microstrip antenna arrays are the narrow frequency band and the mutual coupling between the basic elements is higher than in the usual antenna arrays [8]-[10].

Mutual coupling between array elements affects the radiation pattern and input impedances. The radiation from one element in the array induces currents on the other elements to a nearby and scatters into the far field. The induced current derived a voltage at the terminals of other elements [11].

2. Background

Conventional microstrip antenna has a metallic patch printed on a thin, grounded dielectric substrate. Although the patch can be of any shape, rectangular patches, as shown in Figure 1 [12], are preferred due to easy calculation and modeling.

Fringing fields have a great effect on the performance of a microstrip antenna. In microstrip antennas the electric filed in the center of the patch is zero. The radiation is due to the fringing field between the periphery of the patch and the ground plane. For the rectangular patch shown in the Figure 2, there is no field variation along the width and thickness. The amount of the fringing field is a function of the dimensions of the patch and the height of the substrate. Higher the substrate, the greater is the fringing field.

Due to the effect of fringing, a microstrip patch antenna would look electrically wider compared to its physical dimensions. As shown in Figure 2, waves travel both in
substrate and in the air. Thus an effective dielectric constant $\varepsilon_{\text{reff}}$ is to be introduced. The effective dielectric constant $\varepsilon_{\text{reff}}$ takes into account both the fringing and the wave propagation in the line.

![Figure 2. Electric field lines (Side View).](image)

The expression for the effective dielectric constant is introduced by A. Balanis [12], as shown in Equation 1.

$$\varepsilon_{\text{reff}} = \frac{\varepsilon_r + 1 + \frac{\varepsilon_r - 1}{2} \left[ 1 + 12 \frac{h}{w} \right]}{2}$$  \hspace{1cm} (1)

The length of the patch is extended on each end by $\Delta L$ is a function of effective dielectric constant $\varepsilon_{\text{reff}}$ and the width to height ratio ($W/h$). $\Delta L$ can be calculated according to a practical approximate relation for the normalized extension of the length [8], as in Equation 2.

$$\frac{\Delta L}{h} = 0.412 \frac{(\varepsilon_{\text{reff}} + 0.3)W}{h} + 0.264$$

$$\frac{(\varepsilon_{\text{reff}} - 0.258)W}{R} + 0.8$$  \hspace{1cm} (2)

![Figure 3. Physical and effective lengths of rectangular microstrip patch.](image)

The effective length of the patch is $L_{\text{eff}}$ and can be calculated as in Equation 3.

$$L_{\text{eff}} = L + 2\Delta L$$  \hspace{1cm} (3)

By using the effective dielectric constant (Equation 1) and effective length (Equation 3), we can calculate the resonance frequency of the antenna $f$ and all the microstrip antenna parameters.

**Cylindrical-Rectangular Patch Antenna**

All the previous work for a conformal rectangular microstrip antenna assumed that the curvature does not affect the effective dielectric constant and the extension on the length. The effect of curvature on the resonant frequency has been presented previously [13]. In this paper we present the effect of fringing field on the performance of a conformal patch antenna. A mathematical model that includes the effect of curvature on fringing field and on antenna performance is presented. The cylindrical rectangular patch is the most famous and popular conformal antenna. The manufacturing of this antenna is easy with respect to spherical and conical antennas.

Effect of curvature of conformal antenna on resonant frequency has been presented by Clifford M. Krowne [13, 14] as:

$$\xi(\Omega_{\text{mn}}) = \frac{1}{2\sqrt{\mu\varepsilon}} \sqrt{\left( \frac{m}{2\pi \rho a} \right)^2 + \left( \frac{n}{2b} \right)^2}$$  \hspace{1cm} (4)

where $2b$ is a length of the patch antenna, $a$ is a radius of the cylinder, $\varepsilon$ represents electric permittivity and $\mu$ is the magnetic permeability as shown in Figure 4.

![Figure 4. Geometry of cylindrical-rectangular patch antenna][9]

Joseph A. et al, presented an approach to the analysis of microstrip antennas on cylindrical surface. In this approach, the field in terms of surface current is calculated, while considering dielectric layer around the cylindrical body. The assumption is only valid if radiation is smaller than stored energy[15]. Kwai et al. [16] gave a brief analysis of a thin cylindrical-rectangular microstrip patch antenna which includes resonant frequencies, radiation patterns, input impedances and $Q$ factors. The effect of curvature on the characteristics of $TM_{10}$ and $TM_{01}$ modes is also presented in Kwai et al. paper. The authors first obtained the electric field under the curved patch using the cavity model and then calculated the far field by considering the equivalent magnetic current radiating in the presence of cylindrical surface. The cavity model, used for the analysis is only valid for a very thin dielectric. Also, for much small thickness than a wavelength and the radius of curvature, only $TM$ modes are assumed to exist. In order to calculate the radiation patterns of cylindrical-rectangular patch antenna. The authors introduced the exact Green’s function approach. Using Equation (4), they obtained expressions for the far zone electric field components $E_\theta$ and $E_\phi$ as a functions of Hankel function of the second kind $H_p^{(2)}$. The input impedance and $Q$ factors are also calculated under the same conditions.

Based on cavity model, microstrip conformal antenna on a projectile for GPS (Global Positioning System) device
is designed and implemented by using perturbation theory
is introduced by Sun L., Zhu J., Zhang H. and Peng X [17].
The designed antenna is emulated and analyzed by IE3D
software. The emulated results showed that the antenna
could provide excellent circular hemisphere beam, better
wide-angle circular polarization and better impedance
match peculiarity.

Nickolai Zheliev introduced a design of a small con-
formal microstrip GPS patch antenna [18]. A cavity model
and transmission line model are used to find the initial
dimensions of the antenna and then electromagnetic
simulation of the antenna model using software called
FEKO is applied. The antenna is experimentally tested and
the author compared the result with the software results. It
was founded that the resonance frequency of the
conformal antenna is shifted toward higher frequencies
compared to the flat one.

The effect of curvature on a fringing field and on the
resonance frequency of the microstrip printed antenna is
studied in [19]. Also, the effect of curvature on the
performance of a microstrip antenna as a function of tem-
perature for TM_{01} and TM_{10} is introduced in [21], [21].

3. Conformal Microstrip Antenna
Array and Mutual Coupling

Conformal microstrip arrays are used to increase the
directivity of the antenna and increase the signal to noise
ratio. Better performance is achieved using arrays. The
radiation pattern is significantly affected using arrays on
a conformal surface to appear as omnidirectional pattern,
which is very useful in aerospace systems [22].

The equations of directivity function of the conformal
microstrip array on a cylinder and the experimental results
of pattern of array of 64 elements are given by M. Knghou
et al. [22]. The coupling between elements is not consid-
ered in [22]. The authors calculated the total electric field
strength for an array of N elements using Equation (5).

\[ E = \sum_{i=1}^{N} E_i e^{-j\phi_i} \]  

where, \( E_i \) represents the field strength of number \( i \) radiator and \( \phi_i \) is the phase of equivalent transversal magnetic current
source of \( N \) radiators.

C. You et al. designed and fabricated a composite an-
tenna array conformed around cylindrical structures [23].
The experimental results showed that the radiation pattern
is strongly dependent on the cylindrical curvature for the
transverse radiation pattern, while the array also exhibits
high side-lobes and wider beamwidth.

Problems associated with Ultra Wide Band (UWB)
antennas as phased array elements discussed in [54]. The
authors introduced various wide bandwidth arrays of an-
tennas that can be conforming. Problems that arise de-
pending on the physical separation of antennas are dis-
cussed. Conformal placement, of an antenna, either as an
individual antenna, or as in an array configuration on any
arbitrary surface, may require very thin antenna. The au-
thors should be processed preferably on flexible substrates
so that the authors will conform to the surfaces without
changing the surface geometry.

A. Sangster and R. Jacobs developed a finite ele-
ment-boundary integral method to investigate the im-
pedance properties of a patch element array for a
microstrip printed antenna conformal on a cylindrical
body [25]. A mutual coupling between elements is also
studied in this paper for its great effect on the impe-
dance properties. Simulation results for mutual coupling
coefficient, S_{12}, for a planar and conformal array are
compared to a measured values and a good agreement is
obtained.

A full-wave analysis of the mutual coupling be-
tween two probe-fed rectangular microstrip antennas
conformed on a cylindrical body is introduced by S. Ke
and K. Wong [26]. The authors calculated the mutual
impedance and mutual coupling coefficient using a
moment of method technique [27], [28]. The numerical
results of mutual impedance and mutual coupling coeffi-
cient are compared to the measured values for the
microstrip antennas conformed on a cylindrical body
with different radius of curvatures.

A comprehensive mathematical model for mutual
impedance and mutual coupling between two rectangu-
lar patches array is introduced by A. Mohammadian et al [29].
The authors replaced each microstrip antenna
element in the array by an equivalent magnetic current
source distributed over a grounded dielectric slab. A
dyadic Green’s function is developed for a grounded
dielectric slab, using the rectangular vector wave func-
tions.

The active reflection performance and active radia-
tion pattern of two elements in the array of microstrip
antenna elements are calculated by S. Chen and R. Iwa-
ta [30]. The authors introduced a mathematical deriv-
ation of radiation pattern and reflection performance
for each element in the array of microstrip antenna. Then by
using the introduced model, the mutual coupling be-
tween the elements in the array is easily calculated.

N. Dodov and P. Petkov explored the mutual cou-
pling between microstrip antennas provoked by the sur-
face wave [31]. Based on the method of moments, the
authors analyze the microwave structure on the
microstrip antenna patch surface. N. Dodov and P.
Petkov conclude that, the influence of surface wave is
not significant in close neighboring resonant elements.

An accurate formula for the coupling between
patch elements is introduced by Z. Qi et al [32]. The
classic formula for mutual coupling based on multi-port
network theory ignores the impedance mismatching
between antenna elements but on the other hand the
introduced formula consider this mismatching between
antenna array elements.

A hybrid method, based on the method of moments,
is introduced to analyze a microstrip antenna conformal
on a cylindrical body by A. Erturk et al [33]. The au-
thors introduced three types of space-domain Green’s
function representations, each accurate and efficient in a
given region of space. Input impedance of various
microstrip antenna conformal on a cylindrical body and
mutual coupling between two elements of the array is introduced and compared to some published results.

4. Input Impedance

The input impedance is defined as “the impedance presented by an antenna at its terminals” or “the ratio of the voltage current at a pair of terminals” or “the ratio of the appropriate components of the electric to magnetic fields at a point”. The input impedance is a function of the feeding position as we will see in the next few lines [19].

To get an expression of input impedance $Z_{in}$ for the cylindrical microstrip antenna, we need to get the electric field at the surface of the patch. In this case, we can get the wave equation as a function of excitation current density $J$ as follow:

$$\frac{1}{\rho^2} \frac{\partial^2 E_\rho}{\partial \rho^2} + \frac{\partial^2 E_\phi}{\partial z^2} + k^2 E_\rho = j\omega \mu J$$  \hspace{1cm} (6)

By solving this Equation, the electric field at the surface can be expressed in terms of various modes of the cavity as [19]:

$$E_\rho (\rho, \phi) = \sum_n \sum_m A_{nm} \psi_{nm}(\rho, \phi)$$ \hspace{1cm} (7)

where $A_{nm}$ is the amplitude coefficients corresponding to the field modes. By applying boundary conditions, homogeneous wave Equation and normalized conditions for $\psi_{nm}$, we can get an expression for $\psi_{nm}$ as shown below:

1. $\psi_{nm}$ vanishes at the both edges for the length $L$:

$$\left. \frac{\partial \psi}{\partial x} \right|_{x=0} = \left. \frac{\partial \psi}{\partial x} \right|_{x=L} = 0$$ \hspace{1cm} (8)

2. $\psi_{nm}$ vanishes at the both edges for the width $W$:

$$\left. \frac{\partial \psi}{\partial y} \right|_{y=-\delta_1} = \left. \frac{\partial \psi}{\partial y} \right|_{y=\delta_1} = 0$$ \hspace{1cm} (9)

3. $\psi_{nm}$ should satisfy the homogeneous wave Equation:

$$\left( \frac{1}{\rho^2} \frac{\partial^2}{\partial \rho^2} + \frac{\partial^2}{\partial z^2} + k^2 \right) \psi_{nm} = 0$$ \hspace{1cm} (10)

4. $\psi_{nm}$ should satisfy the normalized condition:

$$\int_{z=0}^{z=L} \int_{\rho=\delta_1}^{\rho=\delta_2} \psi_{nm}^* \psi_{nm} \, d\rho \, dz = 1$$ \hspace{1cm} (11)

Hence, the solution of $\psi_{nm}$ will take the form shown below:

$$\psi_{nm}(\rho, \phi) = \frac{i e^{j \beta_{nm} L}}{2a_\delta L} \cos \left( \frac{\pi}{2} \left( \frac{\rho}{a_\delta} - 1 \right) \right) \cos \left( \frac{\pi}{L} z \right)$$ \hspace{1cm} (12)

with

$$\beta_{nm} = \sqrt{\frac{\varepsilon_{ph}}{2a_\delta}} \left( \varepsilon_0 - \varepsilon_1 \right)$$

The coefficient $A_{nm}$ is determined by the excitation current. For this, substitute Equation (12) into Equation (6) and multiply both sides of (6) by $\psi_{nm}^*$, and integrate over area of the patch. Making use of orthonormal properties of $\psi_{nm}$, one obtains:

$$A_{nm} = \frac{\mu_0}{k^2} \int_{\text{feed}} \psi_{nm}^* \psi_{nm} \, d\rho \, dz$$ \hspace{1cm} (13)

Now, let the coaxial feed as a rectangular current source with equivalent cross-sectional area $S_x \times S_y$ centered at $(Z_0, \phi_0)$, so, the current density will satisfy the Equation below:

$$J_\rho = \begin{cases} \frac{i e^{j \beta_{nm} L}}{2a_\delta L} \cos \left( \frac{\pi}{2} \left( \frac{\rho}{a_\delta} - 1 \right) \right) \cos \left( \frac{\pi}{L} z \right) & \text{if } Z_0 - \frac{S_x}{2} \leq x \leq Z_0 + \frac{S_x}{2}, \\
0 & \text{elsewhere} \end{cases}$$ \hspace{1cm} (14)

Use of Equation (14) in (13) gives:

$$A_{nm} = \frac{\mu_0}{k^2} \int_{\text{feed}} \psi_{nm}^* \psi_{nm} \, d\rho \, dz$$ \hspace{1cm} (15)

So, to get the input impedance, one can substitute in the following Equation:

$$Z_{in} = \frac{V_{in}}{I_0}$$ \hspace{1cm} (16)

where $V_{in}$ is the RF voltage at the feed point and defined as:

$$V_{in} = -E_\rho (Z_0, \phi_0) \times h$$ \hspace{1cm} (17)

By using Equations (7), (12), (14), (17) and substitute in (16), we can obtain the input impedance for a rectangular microstrip antenna conformal in a cylindrical body as in the following Equation:

$$Z_{in} = \frac{j \omega \mu_0}{i} \int_{\text{feed}} \sum_n \sum_m 2a_\delta L \cos \left( \frac{\pi}{2} \left( \frac{\rho}{a_\delta} - 1 \right) \right) \cos \left( \frac{\pi}{L} z \right)$$ \hspace{1cm} (18)
5. Mutual Coupling

Mutual coupling between array elements affects the radiation pattern and input impedances. The radiation from one element in the array induces currents on the other elements near and scatters into the far field. The induced current derived a voltage at the terminals of other elements [11].

The input terminals of the elements in an array are represented as ports of a microwave network. The equivalent network of two antenna array is shown in Figure 5. Hence, the mutual coupling is represented as a scattering matrix or S-parameters matrix as illustrated in Equation 19.

\[
\begin{pmatrix}
 b_1 \\
 b_2 \\
 a_1 \\
 a_2 \\
\end{pmatrix} = \begin{pmatrix}
 S_{11} & S_{12} \\
 S_{21} & S_{22} \\
\end{pmatrix} \begin{pmatrix}
 a_1 \\
 a_2 \\
\end{pmatrix}
\]

(19)

\[
\begin{pmatrix}
 v_1 \\
 v_2 \\
\end{pmatrix} = \begin{pmatrix}
 Z_{11} & Z_{12} \\
 Z_{21} & Z_{22} \\
\end{pmatrix} \begin{pmatrix}
 i_1 \\
 i_2 \\
\end{pmatrix}
\]

(20)

Hence, the mutual coupling coefficient, $S_{12}$, can be calculated as in Equation (21) [26].

\[
S_{12} = 20 \log \left( \frac{2Z_{21}-Z_0}{(Z_{11}+Z_0)^2-(Z_0)^2} \right) \ \text{dB}
\]

(21)

where $Z_0$ is the characteristic impedance of the feeding coaxial cable (assumed to be 50 $\Omega$ in most of the cases).

In case of using identical array elements, same dimension and same feeding position, the values of $Z_{11}$ and $Z_{22}$ will give the same value and $Z_{12}$ and $Z_{21}$ are the same.

Hence, the value of $Z_{12}$ and $Z_{21}$ are given by Equation (22).

\[
Z_{12} = Z_{21} = -\frac{1}{l_0} \int_a^{a+h} E_\rho \left( z_0, \varphi_0, \rho \right) d\rho
\]

(22)

and the value of $Z_{11}$ and $Z_{22}$ are given by Equation (23).

\[
Z_{11} = Z_{22} = Z_{in}
\]

(23)

By using Equations (15) and (18), we can get Equation (24) for $Z_{21}$, as follow:

\[
Z_{21} = Z_{12} = j\omega h \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{k_m^2 - k_n^2} \frac{\varepsilon_m \varepsilon_n}{2a \theta_1 L} \left( \cos^2 \left( \frac{2\pi a m}{L} \left( \frac{d+L}{a} \right) \right) \cos^2 \left( \frac{\pi a}{L} \left( \frac{d+L}{a} \right) \right) \times \right)
\]

\[
\text{sinc} \left( \frac{2\pi a m}{L} \left( \frac{d+L}{a} \right) \right) \text{sinc} \left( \frac{\pi a}{L} \left( \frac{d+L}{a} \right) \right)
\]

(24)

and by substitute in Equation (21) the mutual coupling coefficient can be calculated.

6. Results

For the range of GHz, the dominant mode is $TM_{01}$ which is the case. Also, for the antenna operates at the range 2.5 GHz for three different substrates we can use the following dimensions: the original length is 41.5 cm, the width is 50 cm and for different lossy substrate we can get the effect of curvature on the effective dielectric constant and the resonance frequency.

Three different substrate materials RT/duroid-5880 PTFE, K-6098 Teflon/Glass and Epsilam-10 ceramic-filled Teflon are used for verifying the new model. The dielectric constants for the used materials are 2.2, 2.5 and 10 respectively with a tangent loss 0.0015, 0.002 and 0.004 respectively.

6.1 RT/duroid-5880 PTFE Substrate

RT/duroid-5880 PTFE material is a flexible material with a dielectric constant 2.1 at low frequencies and almost 2.02 in the Giga Hertz range and tangent loss 0.0015.

Return loss (S11) is illustrated in Figure 6 [34]. We obtain a return loss, -36 dB, at frequency 2.1563 GHz for radius of curvature 20 mm, 2.158 GHz at 65 mm and 2.1595 GHz for a flat antenna.
Figure 7. Mutual coupling coefficient, S12, as a function of resonance frequency for different values of curvatures for $TM_{01}$ mode.

Figure 7 shows the mutual coupling coefficient, S12 as a function of resonance frequency for different radius of curvature. The maximum mutual coupling is obtained at the minimum return loss for the same resonance frequency and the peaks are shifted to the direction of increasing frequency with increasing the radius of curvature. The peaks are almost the same at -8 dB, so changing the curvature does not change the mutual coupling value but shift the curve in frequency.

### 6.2 K-6098 Teflon/Glass Substrate

A K-6098 Teflon/Glass material is a flexible material with a dielectric constant 2.5 at high frequency and tangent loss 0.002.

Return loss (S11) is illustrated in Figure 8. We obtain a very low return loss, -50 dB, at frequency 1.935 GHz for radius of curvature 20 mm, 1.937 GHz at 65 mm and 1.938 GHz for a flat antenna. The return loss value, -50 dB, is obtained for different radius of curvature.

Figure 8. Return loss (S11) as a function of frequency for different radius of curvatures.

### 6.3 Epsilam-10 Ceramic-filled Teflon Substrate

Epsilam-10 ceramic-filled Teflon is used as a substrate material for verifying the new model. The dielectric constant for the used material is 10 with a tangent loss 0.004.

Return loss (S11) is illustrated in Figure 10. We obtain a return loss, -4.3 dB for all values of radius of curvature, 20 mm, 65 mm and flat antenna.

Figure 10. Return loss (S11) as a function of frequency for different radius of curvatures.
7. Conclusion

The effect of curvature on the performance of conformal microstrip antenna on cylindrical bodies for TM\textsubscript{01} mode is very important. Curvature affects the fringing field and fringing field affects the antenna parameters. The Equations for return loss and mutual coupling coefficient as a function of curvature and resonance frequency are derived.

By using these derived equations, we introduced the results for different dielectric conformal substrates. For the three dielectric substrates, the decreasing in frequency due to increasing in the curvature is the trend for all materials. We conclude that, increasing the curvature leads to decreasing the resonance frequency. The return loss peaks do not change for all substrate materials, but the mutual coupling coefficient peaks are changing according to the substrate material used.

REFERENCES


